

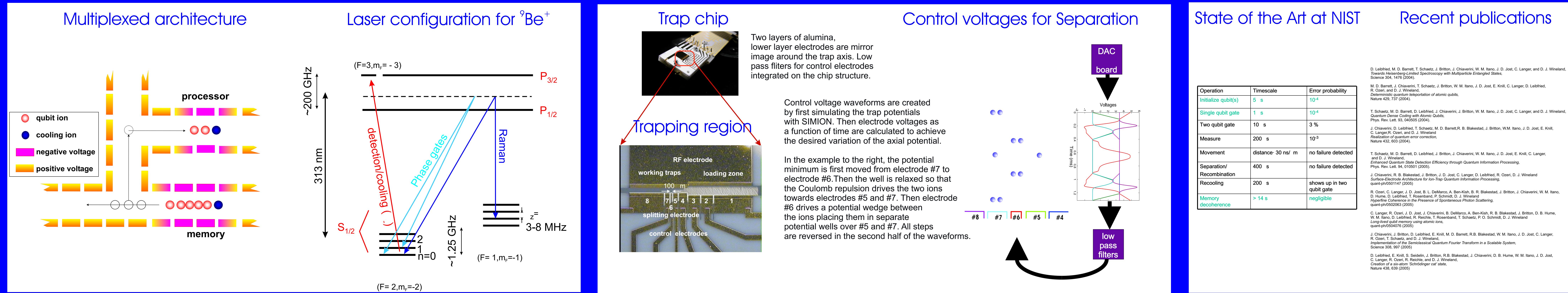
Small quantum algorithms realized in a scalable ion trap array

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General approach



GHZ-states of up to 6 ions

GHZ-state generation and entanglement detection

GHZ states for 3,4 and 5 ions of the form

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\dots\uparrow\rangle + |\downarrow\downarrow\dots\downarrow\rangle)$$

are produced by sandwiching an appropriate phase gate in between $\frac{1}{2}$ -pulses. Upon free precession these states pick up phase N -times faster than a single ion (top trace). This makes such states useful for spectroscopy beyond the standard limit.

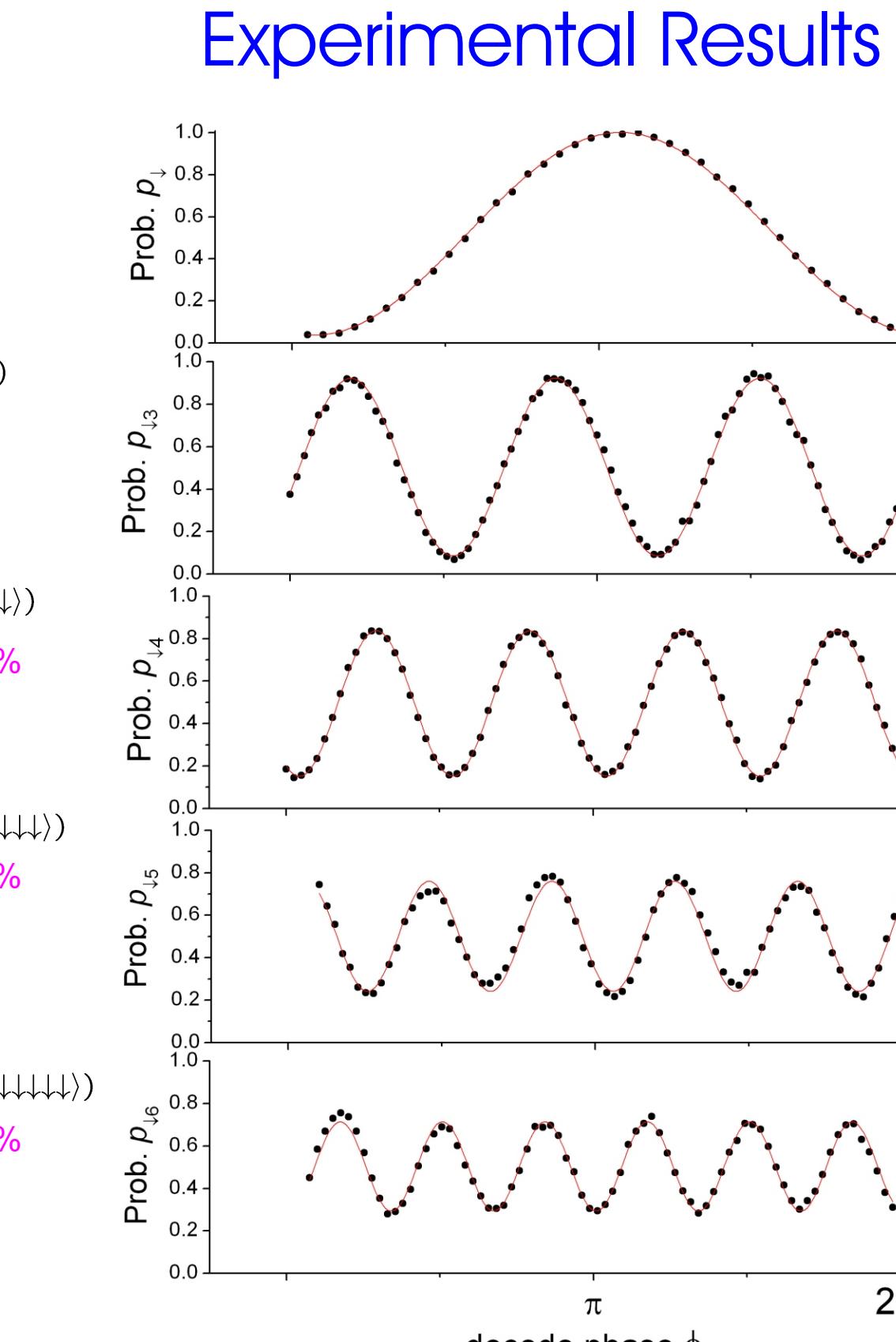
To prove genuine N -particle entanglement, one can use the witness operator

$$\hat{W}(N) = \hat{I}/2 - |\text{GHZ}\rangle\langle\text{GHZ}|$$

When the contrast of the phase sweep is larger than 0.5 the expectation value of the witness is negative by the inequality

$$\begin{aligned} \text{Tr}(\hat{\rho}(N)\hat{W}(N)) &= 1/2 - F \\ &= 1/2 - 1/2(P_{N\uparrow} + P_{N\downarrow}) - \rho_{N\uparrow N\downarrow} \\ &\leq 1/2 - 2\rho_{N\uparrow N\downarrow} = 1/2 - \text{contrast} \end{aligned}$$

The contrast of the phase sweep is a lower bound for fidelity since the gate that produces the GHZ state is used a second time to "decode" the phase information. This implies that the fidelity of our GHZ states could be as high as the square-root of the contrast.



Three Qubit Bitflip-Error Correction

The Code

Our implementation is based on a 3-qubit stabilizer code specially developed for easy implementation with a **single phase gate**. The code has no classical analog and is therefore not locally equivalent to the repetition code or a CSS code.

The code is based on the generators (ZZX, ZXZ) and can either detect or correct within several different correction protocols. In our experiment we corrected for bit-flip (X) errors on all three qubits.

Check matrices for some protocols

Error detection (X-errors on qubit 1, all errors on 2,3)	III	XII	IIX	IIX	IIZ
	0	1	0	1	0
	ZZX	0	1	0	1
Error correction (X-errors on all qubits)	III	XII	IIX	IIX	IIZ
	0	1	0	1	0
	ZZX	0	1	0	1
Error correction (X-errors on qubit 1, Z-errors on 2,3)	III	XII	IIZ	IIZ	IIZ
	0	1	0	1	0
	ZZX	0	1	0	1
Error correction (X-errors on qubit 1 and 2, Z-errors on 2)	III	XII	IIZ	IIX	IIZ
	0	1	0	1	0
	ZZX	0	1	0	1
etc.					

Experimental Results

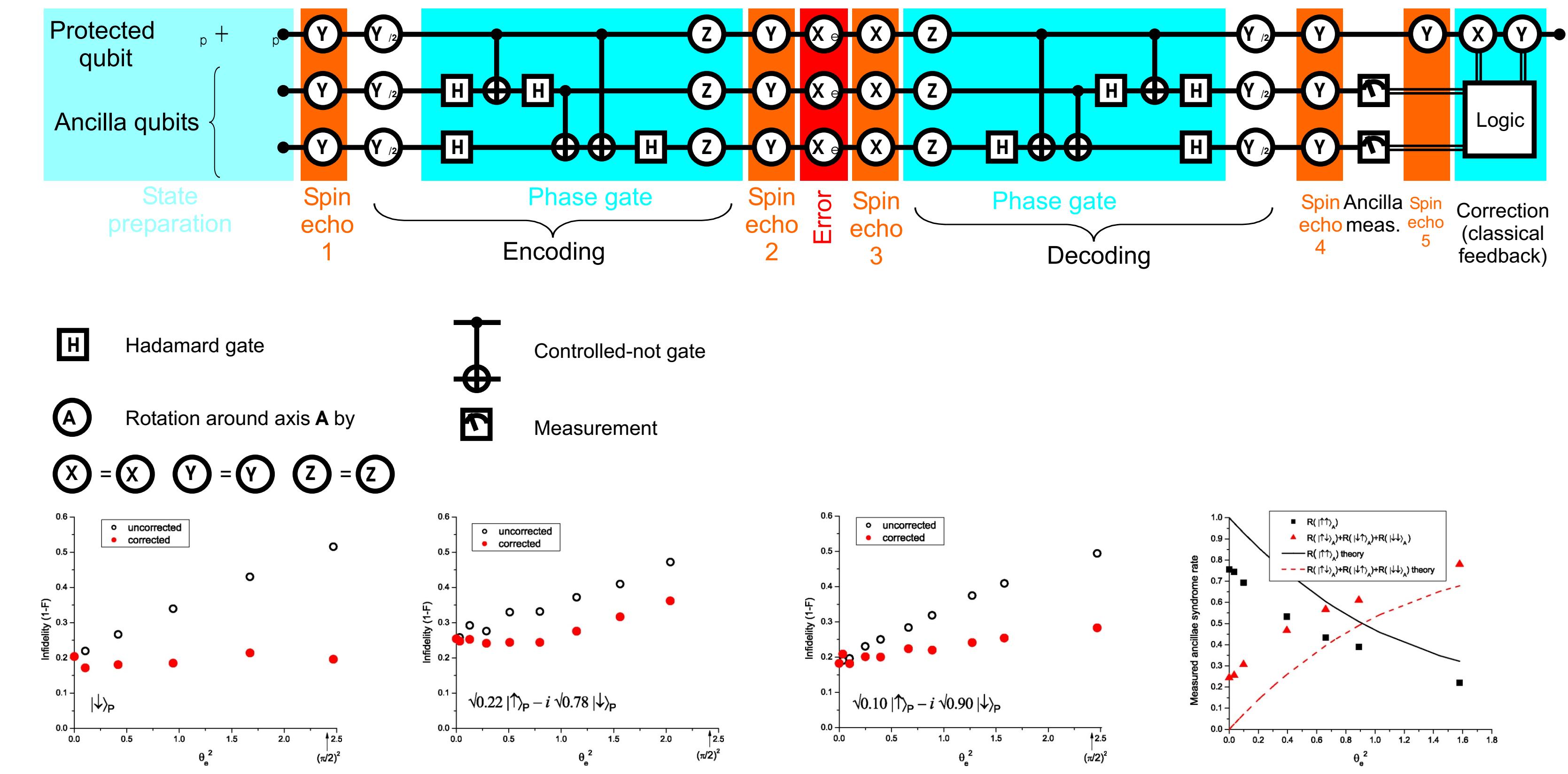
We encoded states of the form $|+\rangle$ and then applied an "error", a controlled pulse with rotation angle θ . This rotation was applied either to the **encoded state** (red symbols) or unencoded state (black symbols). For small angles the infidelity should scale as θ^2 , therefore the black curve is roughly linear. For the encoded state the linear error is suppressed by the error correction, therefore it starts out quadratically in θ .

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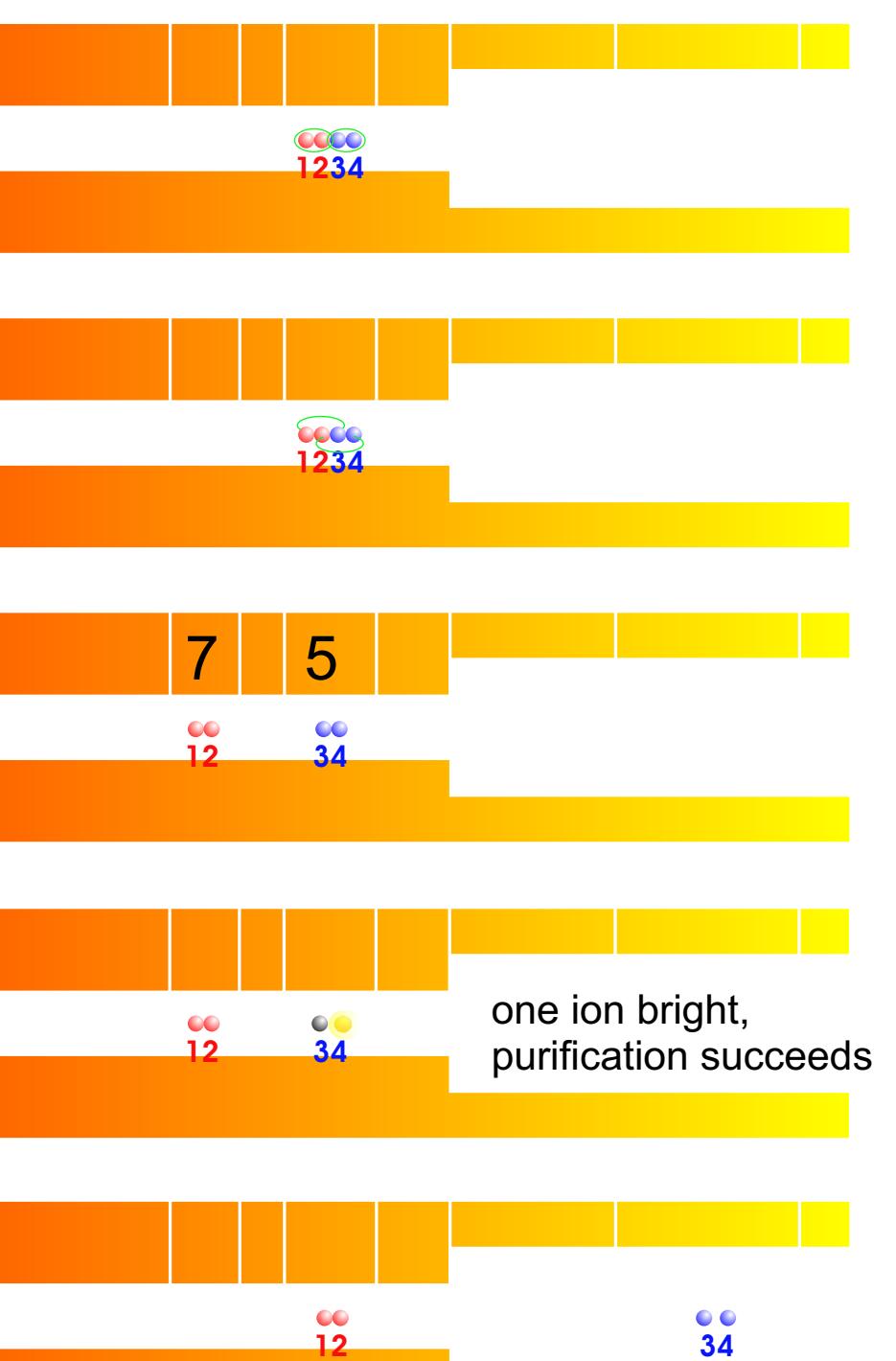
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Experimental Implementation

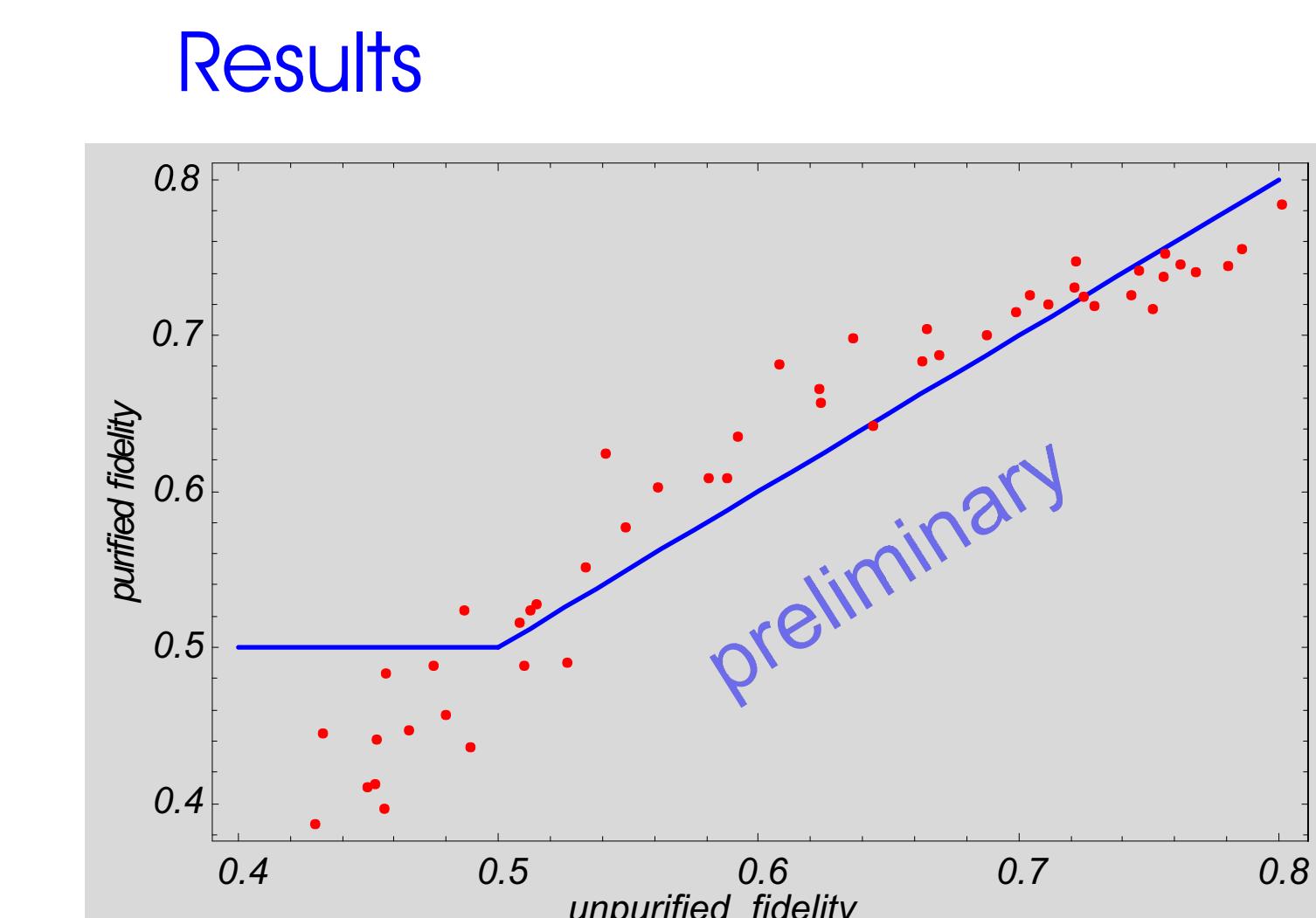


Entanglement Purification (Distillation)

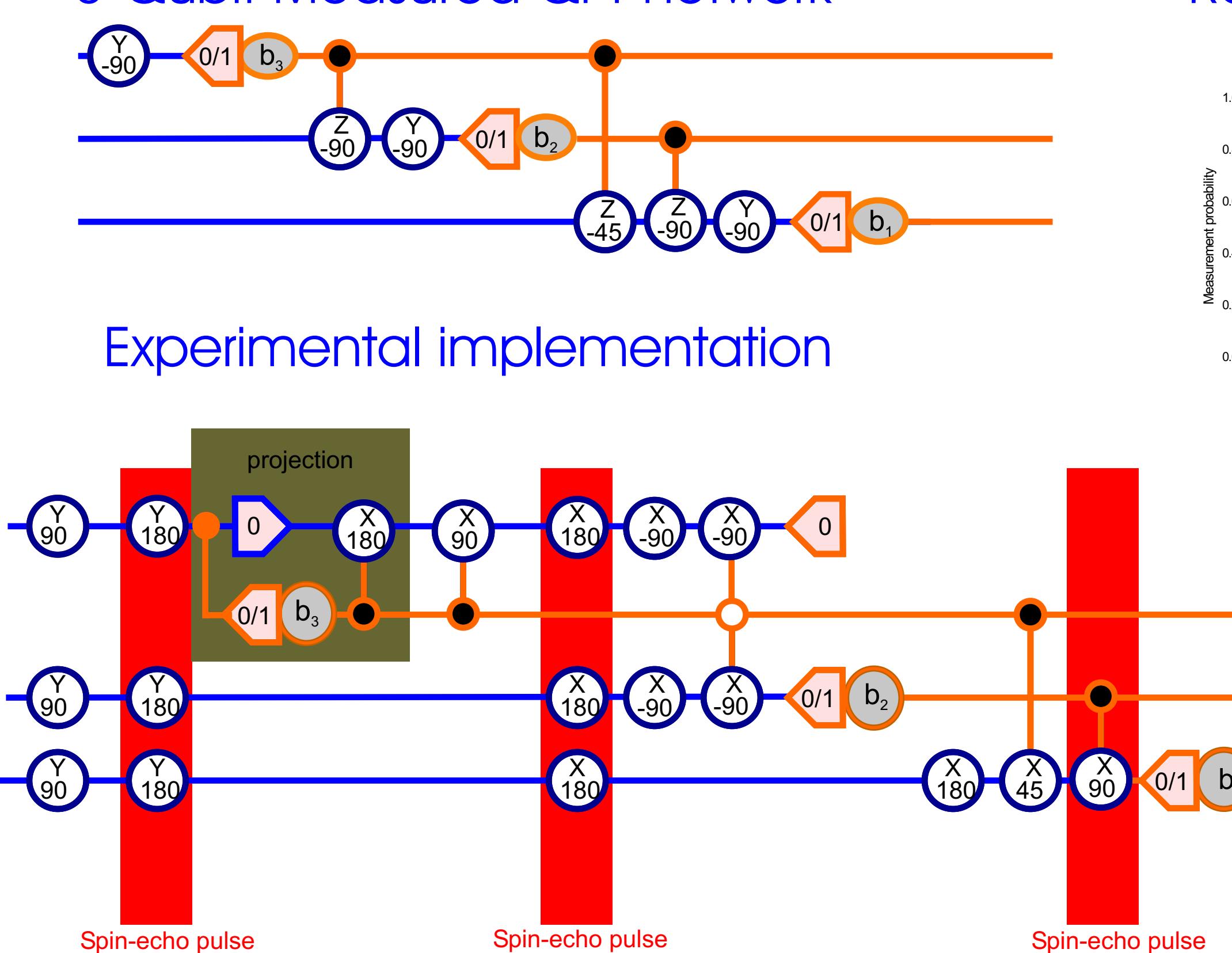
Experimental implementation



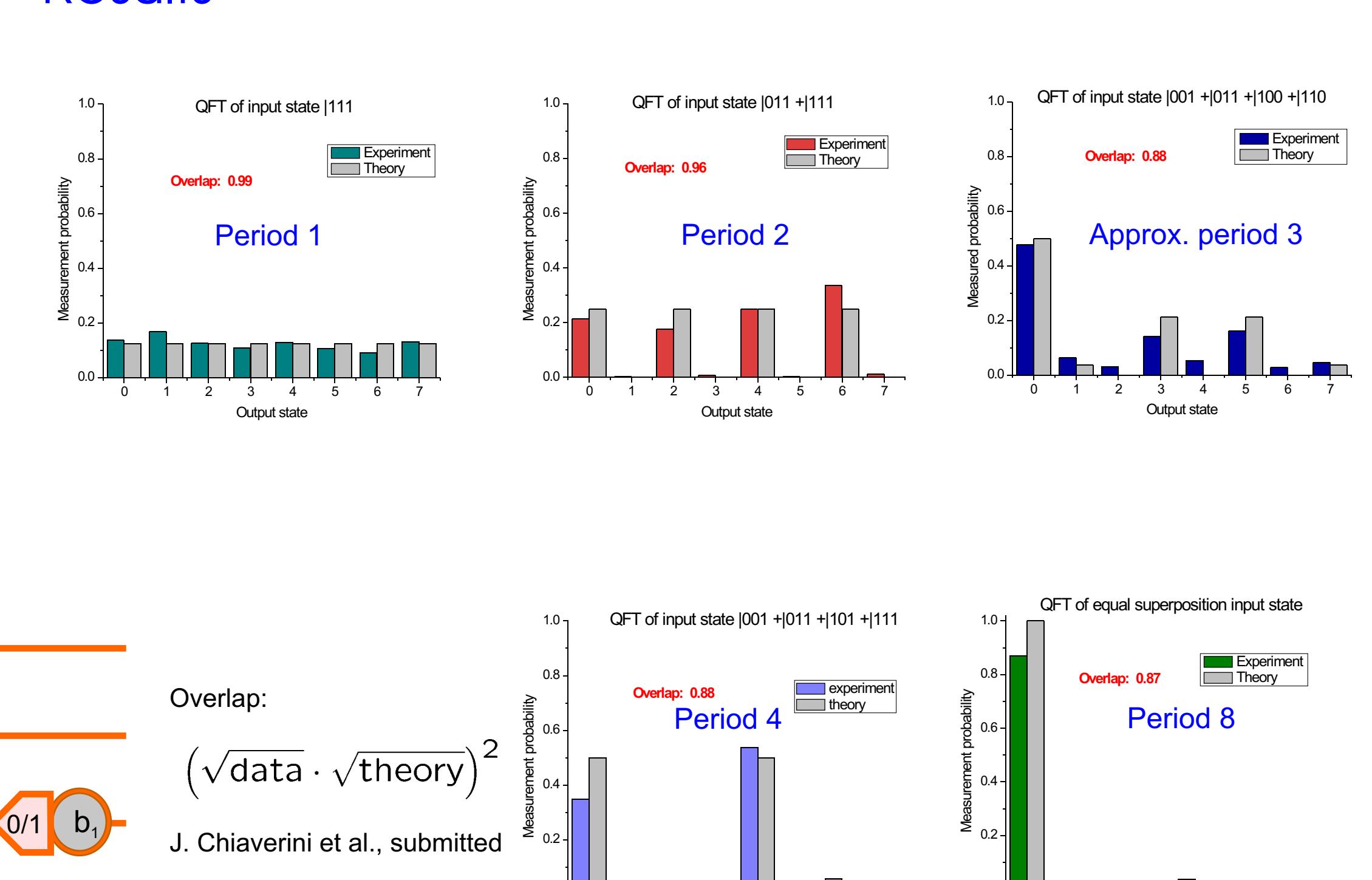
Results



3-Qubit Measured QFT network



Results



Experimental implementation

